S–2999

Sub. Code
23MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

ALGEBRAIC STRUCTURES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Determine $P(5)$.

2. Define $P-$sylow subgroup of G.

3. When will you say that a group G is said to be solvable?

4. Define $R-$module with an example.

5. What is meant by similarity class?

6. Is the integers $n_1, n_2, \ldots n_r$ are said to be invariants of T? Justify.

7. Define Jordan canonical form.

8. Define the rational canonical form of T.

9. If A is invertible then prove that $tr\,(ACA^{-1}) = tr\,c$.

10. Define the following terms :

(a) Skew – symmetric matrix

(b) Skew – Hermitian matrix

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If G is a finite group, then prove that $C_a = \dfrac{O(G)}{O(N(a))}$.

Or

(b) If $O(G) = P^n$ where P is a prime number, then prove that $Z(G) \neq (e)$.

12. (a) With the usual notations, prove that S_n is not solvable for $n \geq 5$.

Or

(b) Let R be a Euclidena ring. Prove that any finitely generated $R-$ module, M is the direct sum of a finite number of cyclic submodules.

13. (a) Prove that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent. Also find its invariants.

Or

(b) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

14. (a) Find all possible Jordan forms for all 8×8 matrices having $x^2(x-1)^3$ as minimal polynomial.

Or

(b) Suppose that T, in $A_F(V)$, has as minimal polynomial over F the polynomial $p(x) = \gamma_0 + \gamma_1 x + \ldots + \gamma_{r-1} x^{r-1} + x^r$. Suppose, further, that V, as a module, is a cyclic module. Prove that there is basis of V over F such that, in this basis, the matrix of T is $\begin{pmatrix} 0 & 1 & 0 & \ldots. & 0 \\ 0 & 0 & 1 & \ldots. & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots. & 1 \\ -\gamma_0 & -\gamma_1 & \ldots & \ldots & -\gamma_{r-1} \end{pmatrix}$.

15. (a) If F is of characteristic O and if S and T, in $A_F(V)$, are such that $ST - TS$ commutes with S, then prove that $ST - TS$ is nilpotent.

Or

(b) Define the normal transformation. If N is normal and $AN = NA$, then prove that $AN^* = N^* A$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If p is a prime number and $\frac{p^\alpha}{O(G)}$, then prove that G has a subgroup of order p^α.

17. Show that two abelian groups of order p^n are isomorphic if and only if they have the some invariants.

18. (a) If $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $o < k \leq n_1$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

(b) Show that two nilpotent linear transformation are similar if and only if they have the same invariants.

19. Prove that for each $i = 1, 2, ..., k, V_i \neq 0$ and $V = V_1 \oplus V_2 \oplus .... \oplus V_k$. The minimal polynomial of T_i is $q_i(x)^{l_i}$.

20. Given the real symmetric matrix A there is an invertible matrix T such that $T\,A\,T' = \begin{pmatrix} I_r & & \\ & -I_s & \\ & & O_t \end{pmatrix}$ where I_r and I_s are respectively the $r \times r$ and $s \times s$ unit matrices and where O_t is the $t \times t$ zero matrix. The integers $r + s$, which is the rank of A, and $r - s$, which is the signature of A, characterize the congruence class of A. Prove that two real symmetric matrices are congruent if and only if they have the same rank and signature.

S–3000

Sub. Code
23MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

REAL ANALYSIS — I

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that a function is said to be bounded variation on $[a, b]$.

2. Define an alternating series.

3. Define a step function.

4. State the Riemann's condition with respect to α on $[a,b]$.

5. What are the conditions of sufficient for the existence of the Riemann integral $\int_a^b f(x)\,dx$ .s

6. State first Mean – value theorem for Riemann – Stieltjes integrals.

7. Define a double sequence.

8. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$.

9. When will you say that a sequence $\{f_n\}$ is said to be uniformly bounded on S ?

10. State the Dirichlet's test for uniform convergence theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows : $V(x) = V_f(a,x)$ if $a < x \le b, V(a) = 0$. Prove that

(i) V is an increasing function on $[a,b]$;

(ii) $V - f$ is an increasing function on $[a,b]$.

Or

(b) State and prove the Abel's test theorem.

12. (a) Assume that $c \in (a,b)$. If two of the three integrals in (1) exist, then prove that the third also exists and

$$\int_a^c f\,d\alpha + \int_c^b f\,d\alpha = \int_a^b f\,d\alpha. \text{———} (1)$$

Or

(b) State and prove the Euler's summation formula.

13. (a) State and prove the second fundamental theorem of integral calculus.

Or

(b) (i) State and prove the Bonnet's theorem.

(ii) Let f be defind and bounded on $[a,b]$. For each $\in > 0$ define the set $J_\in$ as follows :

$J_\in = \{x : x \in [a, b], w_f(x) \ge \in\}$. Prove that $J_\in$ is a closed set.

14. (a) Assume that each $a_n \geq 0$. Prove that the product $\pi(1-a_n)$ converges if and only if the series $\sum a_n$ converges.

Or

(b) State and prove the Tauber theorem.

15. (a) State and prove that Weierstrass M – test theorem.

Or

(b) Assume that $\lim_{n\to\infty} f_n = f$ on $[a,b]$. If $g \in R$ on $[a,b]$, define $h(x) = \int_a^x f(t)\ g(t)\ dt,\ h_n(x) = \int_a^x f_n(t)\ g(t)\ dt$. If $x \in [a, b]$. Prove that $h_n \to h$ uniformly on $[a,b]$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Define an absolutely convergent of a series. Also prove that absolute convergence of $\sum a_n$ implies convergence.

(b) Let $\sum a_n$ be an absolutely convergent series having sum s. Prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum s.

17. Assume that $\alpha \nearrow$ on $[a,b]$. Prove the following three statements are equivalent:

(a) $f \in R(\alpha)$ on $[a,b]$.

(b) f satisfies Riemann's condition with respect to α on $[a, b]$.

(c) $\underline{I}\ (f, \alpha) = \overline{I}\ (f, \alpha)$.

18. Assume that α is of bounded variation on $[a,b]$. Let $V(x)$ denote the total variation of α on $[a, x]$ if $a < x \leq b$, and let $V(a) = 0$. Let f be defined and bounded on $[a, b]$. Then prove that $f \in R(V)$ on $[a,b]$.

19. State and prove the Bernstein theorem.

20. Assume that each term of $\{f_n\}$ is a real – valued function having a finite derivative at each point of an open interval (a,b). Assume that for at least one point x_0 in (a,b) the sequence $\{f_n(x_0)\}$ converges. Assume further that there exists a function of such that $f_n^1 \to g$ uniformly on (a,b). Prove that

    (a) There exists a function f such that $f_n \to f$ uniformly on (a,b).

    (b) For each x in (a,b) the derivative $f'(x)$ exists and equals $g\ (x)$.

S–3001

Sub. Code
23MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find all solutions φ of $y'' + y = 0$ satisfying $\varphi(0) = 0$, $\varphi(\pi) = 0$.

2. Compute the Wronskian of $\varphi_1(x) = x^2$, $\varphi_2(x) = 5x^2$.

3. State Existence theorem for linear equation with constant coefficients.

4. Determine all real-valued solutions of the equation $y^{(4)} - y = 0$.

5. Write down the following equations:

   (a) Chebyshev; (b) Hermite.

6. Show that $P_n(-x) = (-1)^n P_n(x)$.

7. Check whether the singular point $x = 1$ of the equation $\left(x^2 + x - 2\right)^2 + 3(x+2)y' + (x-1)y = 0$ is regular or not?

8. State the Laguerre equation and n^{th} Laguerre polynomial.

9. Show that the function f given by $f(x,y) = 4x^2 + y^2$ satisfy Lipschitz condition on $S : |x| \leqq 1$, $|y| \leqq 1$.

10. Determine whether the equation $x^2 y^3 dx - x^3 y^2 dy = 0$ is exact or not.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let φ_1, φ_2 be two solutions of $L(y) = 0$ on an interval I, and let x_0 be any point in I. Prove that φ_1, φ_2 are linearly independent on I if and only if $W(\varphi_1, \varphi_2)(x_0) \neq 0$.

Or

(b) Let $L(y) = y'' + a_1 y' + a_2 y$, where a_1, a_2 are constants and let p be the characteristic polynomial $p(r) = r^2 + a_1 r + a_2$.

(i) If A, α are constants and $p(\alpha) \neq 0$, show that there is a solution φ of $L(y) = Ae^{\alpha x}$ of the form $\varphi(x) = Be^{\alpha x}$, where B is constant.

(ii) Compute a particular solution of $L(y) = Ae^{\alpha x}$ in case $p(\alpha) = 0$.

12. (a) Consider the equation $y''' - 4y' = 0$.

(i) Compute three linearly independent solutions.

(ii) Compute the Wronskian of the solutions found in (i).

Or

(b) Find all solutions of the equation $y'' - 2iy' - y = e^{ix} - 2e^{-ix}$.

13. (a) Prove that there exists n linearly independent solution of $L(y)=0$ on I.

Or

(b) Find two linearly independent power series solutions (in powers of x) of the equation $y'' + x^3 y' + x^2 y = 0$.

14. (a) Compute the indicial polynomial and their roots of the equation $x^2 y'' + (\sin x) y' + (\cos x) y = 0$.

Or

(b) Obtain two linearly independent solution of the following equation which are valid near $x = 0$:

$$x^2 y'' - 2x(x+1) y' + 2(x+1) y = 0 .$$

15. (a) Solve the equation $y' = \dfrac{x+y+1}{2x+2y-1}$.

Or

(b) Consider the problem $y' = 1 - 2xy$, $y(0) = 0$. Since the differential equation is linear, an expression can be found for the solution. Find it.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let φ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Prove that for all x in I, $\|\varphi(x_0)\| e^{=k}|x - x_0| \leqq \|\varphi(x)\| \leqq \|\varphi(x_0)\| e^{k|x-x_0|}$ where $\|\varphi(x)\| = \left[|\varphi(x)|^2 + |\varphi'(x)|^2\right]^{1/2}$, $k = 1 + |a_1| + |a_2|$.

17. (a) Using the annihilator method find a particular solution of $y'' + 9y = x^2 e^{3x}$.

(b) Show that if g has k derivatives and r is a constant, $D^k\left(e^{rx} g\right) = e^{rx}(D+r)^k(g)$.

18. Prove the following

(a) $\int_{-1}^{1} P_n(x) P_m(x) dx = 0$, $(n \neq m)$.

(b) $\int_{-1}^{1} P_n^2(x) dx = \frac{2}{2n+1}$.

19. Derive the Bessel function of order α of the first kind.

20. (a) Show that a function φ is a solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_2}^{x} f(t,y) dt$ on I.

(b) Find an integrating factors of the equation $\left(2y^3 + 2\right)dx + 3xy^2 dy = 0$.

S–3002

Sub. Code
23MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective – NUMBER THEORY AND CRYPTOGRAPHY

(CBCS – 2023 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write any two properties of divisibility.

2. Convert 9679_{10} to binary form.

3. Find the canonical factorization of 2646.

4. Define composite number with an example.

5. State Chinese remainder theorem.

6. Define congruence with an example.

7. Evaluate $\phi(735)$.

8. Find the value of $\tau(360)$.

9. Write down the aftline map with an example.

10. Define discrete log.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove Binomial theorem.

Or

(b) Find x, y such that $(418,165) = 418x + 165y$. Also determine [418, 165].

12. (a) Prove that there exists infinitely many primes.

Or

(b) If d and n are natural numbers then prove that $\left[\frac{n}{d}\right] - \left[\frac{n-1}{d}\right] = \begin{cases} 1 & if \quad d \mid n \\ 0 & if \quad d \nmid n \end{cases}$.

13. (a) If P is a prime and $(a, p) = 1$ then prove that $a^{P=1} \equiv 1 (\bmod P)$.

Or

(b) Prove that the congruence $ax \equiv b (\bmod m)$ has at least one solution if and only if $(a, m) \mid b$.

14. (a) Evaluate (i) $\left(\frac{-35}{97}\right)$; (ii) $\left(\frac{71}{73}\right)$; (iii) $\left(\frac{8}{71}\right)$.

Or

(b) State and prove Moebius inversion formula.

15. (a) Write down the applications of public key cryptography.

Or

(b) Find the inverse of the following matrix $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix} (\bmod 5)$. Write the entries in the inverse matrix as non-negative integers less than 5.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let $\alpha = \frac{1+\sqrt{5}}{2}$. If a and b are integers such that $a > b > 0$ and n is the number of iterations needed to compute (a,b) using Euclid's Algorithm. Then prove that $n \le 1 + \log_{\alpha} b$.

17. State and prove the fundamental theorem of Arithmetic.

18. State and prove Wilson's theorem.

19. State and prove lemma of Gauss.

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(Z/NZ)$ and $D = ad - bc$ prove that the following are equivalent:

    (a) g.c.d. $(D, N) = 1$

    (b) A has an inverse matrix.

    (c) If x and y are not both O in Z/NZ, then $A\begin{pmatrix} x \\ y \end{pmatrix} \ne \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

    (d) A gives a 1-to-1 correspondence of $(Z/NZ)^2$ with itself.

S–3003

Sub. Code
23MMA1E2

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective – GRAPH THEORY AND APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the adjacency matrix and the incidence matrix of graph G with an example for each.

2. What is meant by a bond?

3. Define a vertex cut of a graph with an example.

4. Draw a Herschel graph.

5. What is meant by matching in graph G?

6. Draw four – edge chromatic graph.

7. Define the Ramsey numbers. Also find $r(2,l)$.

8. Write down the Hajor conjecture with an example.

9. If G is a simple planer graph, then prove that $\delta \leq 5$.

10. How many orientations does a simple graph G have?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) (i) Prove that in any graph, the number of vertices of odd degree is even.

(ii) With the usual notations, prove that $\delta \leq 2E/\upsilon \leq \Delta$.

Or

(b) State Cayley's formula. If e is a ink of G, then prove that $\tau(G) = \tau(G-e) + \tau(G.e)$.

12. (a) Define a block of a graph with an illustration. If G is a block with $\upsilon \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

13. (a) Prove that every three-regular graph without cut edges has a perfect matching.

Or

(b) If G is bipartite graph, then prove that $\chi' = \Delta$.

14. (a) Define an independent set of graph G with an example. Also prove that $\alpha' + \beta' = \upsilon$ if $\delta > 0$.

Or

(b) If G is four – chromatic, then prove that G contains a subdivision of K_4.

15. (a) Let υ be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that υ is an the exterior face of the embedding.

Or

(b) Draw the graph of all the tournaments on four vertices. Also prove that every tournament has a directed Hamilton path.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Prove that a graph is biparticle if and only if it contains no odd cycle.

17. (a) If G is a simple graph with $\upsilon \geq 3$ and $\delta \geq \upsilon/2$, then prove that G is Hamiltonian.

(b) Show that $C(G)$ is well defined.

18. State and prove the Vizing's theorem.

19. For any positive integer k, prove that there exists a k-chromatic.

20. Prove that every planar graph is five-vertex – colourable.

———————

S–3004

Sub. Code
23MMA1E3

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective – FORMAL LANGUAGES AND AUTOMATA THEORY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Draw the transition diagram of a finite Automaton.

2. Construct left-linear and right-linear grammers for the language 0(10)*

3. Show that $L = \{O^{2n} \mid n \geq 1\}$ is not regular.

4. Name any four closure properties of regular language.

5. What is meant by GNF?

6. Let G be the grammar $S \to ab \mid ba$, $A \to a \mid as \mid bAA$, $B \to b \mid bs \mid aBB$. For the string $aaabbabbba$. Find a deviation which is neither leftmost nor rightmost.

7. Define Instaneous description.

8. What is meant by deterministic *PDA'S*.

9. State ogden's lemma.

10. Define Turning machine.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_0\})$ where δ is given by $\delta(q_0,0) = q_2$; $\delta(q_0,1) = q_1$; $\delta(q_1,0) = q_3$; $\delta(\mathrm{q}_1,1) = \mathrm{q}_0$; $\delta(q_2,0) = q_0$; $\delta(q_2,1) = q_3$; $\delta(q_3,0) = q_1$; $\delta(q_3,1) = q_2$.

(i) Draw the transition diagram.

(ii) Is the string 110101 in $L(M)$?

Or

(b) Construct *DFD'S* equivalent to the *NFA'S* $M = (\{p, q, r, s\}, \{0,1\}, \delta_2, p, \{q, s\})$ where δ_2 is given by

	0	1
p	q,s	q
q	r	q,r
r	s	p
s	–	p

12. (a) Show that the class of regular sets is closed under homomorphisms and inverse homomorphisms.

Or

(b) State and prove pumping lemma for regular sets.

13. (a) Let G be a grammar with $S \to Aas \mid a, A \to SbA \mid ss \mid ba$. For the string *aabbaaa*. Find

(i) left most deviation

(ii) parse tree.

Or

(b) Reduce the grammar G to CNF given that $S \to \sim S \mid [S \supset S] \mid p \mid q$ are the productions in G.

14. (a) Construct a *PDA* equivalent to the following context t-free grammar $S \to aBB, B \to as, B \to bs, B \to a$.

Or

(b) Construct a *CFG* accepting $N(M)$ where

$M = (\{q_0, q_1\}, \{0,1\}, \{x, z_0\}, \delta, q_0, z_0, \phi)$, where δ is given by $\delta(q_0, 0, z_0) = \{(q_0, x\, z_0)\}$, $\delta(q_1, 1, x) = \{(q_1, \in)\}$,

$\delta(q_0, 0, x) = \{(q_0, xx)\}$, $\delta(q_1, \in, x) = \{(q_1, \in)\}$,

$\delta(q_0, 1, x) = \{(q_1, \in)\}$, $\delta(q_1, \in, z_0) = \{(q_1, \in)\}$.

15. (a) State and prove pumping Lemma for *CFL'S*.

Or

(b) Show that the context - free languages are closed under substitution.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If L is the set accepted by *NDFA*, then prove that there exists a *DFA* which also accepts L.

17. State and prove Myhill - Nerode theorem.

18. Prove that any context - free language without $\in$ is generated by a grammar in which all productions are of the form $A \rightarrow BC$ or $A \rightarrow a$ where A, B, C are variables and a is a terminal.

19. Prove that there exists a context free grammar G such that $N(M) = L(G)$ where $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is a PDA.

20. Design turning Machines to compute the n!.

S–3005

Sub. Code
23MMA1E4

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective — MATHEMATICAL PROGRAMMING

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is integer programming?

2. Define mixed integer programming problem.

3. What difficulties you overcome when dynamic programming is designed?

4. State Bellman's principle of optimality.

5. What is goal programming?

6. Write the mathematical formation of GP.

7. Define NLPP.

8. State Kuhn – Tuker conditions in NLPP.

9. Define Pseudo random number.

10. List any two areas of application of simulation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Solve the following mixed – integer programming problem

Max $z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 5$; $x_2 \leq 2$ and $x_1, x_2 \geq 0, x_1$ non-negative integer.

Or

(b) Explain Branch and bound method for solving an integer programming problem.

12. (a) Use dynamic programming to find the value of

Max $z = u_1 \; u_2 \; u_3$

Subject to $u_1 + u_2 + u_3 = 10$ and $u_1, u_2, u_3 \geq 0$.

Or

(b) Solve the following Linear programming problem

Max $z = 3x_1 + 5x_2$

Subject to $x_1 \leq 4; x_2 \leq 6; 3x_1 + 2x_2 \leq 18$ and $x_1, x_2 \geq 0$.

13. (a) Explain the Ranking and weighting of unequal multiple goals.

Or

(b) Explain the graphical solution method for solving a Goal programming.

14. (a) Write short notes on Application of quadratic programming problem.

Or

(b) Solve graphically the following NLP problem

Max $z = 2x_1 + 3x_2$

Subject to $x_1^2 + x_2^2 \leq 20$; $x_1 \ x_2 \leq 8$ and $x_1, x_2 \geq 0$.

15. (a) A bakery keeps stock of a popular brand of cake, previous experience shows the daily demand pattern for the item with associated probabilities, as given below :

Daily demand (number) :	0	10	20	30	40	50
Probability :	0.01	0.20	0.15	0.50	0.12	0.02

Use the following sequence of random number to simulate the demand for next 10 days.

Random numbers : 25, 39, 65, 76, 12, 05, 73, 89, 19, 49. Also estimate the daily average demand for the cakes on the basis of simulated data.

Or

(b) A firm has a single channel service station with the following arrival and service time probability distributions :

Interarrival Time (Minutes)	Probability	Service Time (Minutes)	Probability
10	0.10	5	0.08
15	0.25	10	0.14
20	0.30	15	0.18
25	0.25	20	0.24
30	0.10	25	0.22
		30	0.14

The customer's arrival at the service station is a random phenomenon and the time between the arrivals varies from 10 minutes to 30 minutes. The service time varies from 5 minutes to 30 minutes. The queuing process begins at 10 a.m. and proceeds for nearly 8 hours. An arrival goes to the service facility immediately, if it is free. Otherwise it will wait in a queue. The queue discipline is first – come first – served. If the attendant's wages are Rs.10 per hour and the customer's waiting time costs Rs.15 per hour, then would it be an economical proposition to engage a second attendant? Answer using Monte Carlo simulation technique.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Solve the following integer linear programming problem using the cutting plane Algorithm. Max $z = 2x_1 + 20x_2 - 10x_3$ subject to $2x_1 + 20x_2 + 4x_3 \leq 15$; $6x_1 + 20x_2 + 4x_3 = 20$ and $x_2, x_3 \geq 0$ and are integers.

17. A student has to take an examination in three courses x, y and z. He has three days available for study. He feels that it would be better to devote a whole day to study of the same course, so that he may study a course for one day, two days or three days or not at all. His estimates of grades he may get according to days of study he puts in are as follows :

Study days	Course		
	x	y	z
0	1	2	1
1	2	2	2
2	2	4	4
3	4	5	4

How should he plan to study so that he maximizes his the sum of grades?

18. Solve the following goal programming problem using the graphical method to obtain the solution

Minimize $z = p_1\, d_1^- + p_2\, d_2^- + p_3\, d_3^-$

Subject to $2x_1 + 3x_2 \le 30$; $6x_1 + 4x_2 \le 60$; $x_1 + x_2 + d_2^- - d_2^+ = 8$; $x_2 + d_3^- - d_3^+ = 7$ and $x_1, x_2, d_i^-, d_i^+ \ge 0$ for all i.

19. Use the wolfe's method to solve the quadratic programming problem

Max $z = 2x_1 + x_2 - x_1^2$.

Subject to $2x_1 + 3x_2 \le 6$; $2x_1 + x_2 \le 4$ and $x_1, x_2 \ge 0$.

20. A project consists of 8 activities A to H. The completion time for each activity is a random variable.

The data concerning probability distribution along with completion times for each activity is a follows:

Activity	Immediate Predecessor(s)	Time (day)/ Probability								
		1	2	3	4	5	6	7	8	9
A	–	–	–	–	0.2	–	0.4	0.4	–	–
B	–	–	–	–	–	–	0.5	–	0.5	–
C	A	–	–	0.7	0.3	–	–	–	–	–
D	B,C	–	–	–	–	0.9	–	–	0.1	–
E	A	–	–	–	–	0.2	–	–	–	0.8
F	E	–	–	–	0.6	0.4	–	–	–	–
G	E	–	–	0.4	0.4	–	0.2	–	–	–
H	F	–	0.4	–	–	–	–	0.6	–	–

(a) Draw the network diagram and identify the critical path using the expected activity times.

(b) Simulate the project to determine the activity times, Determine the critical path and project expected completion time.

(c) Repeat the simulation four times and state estimated duration of the project in each of the trails.

S–3006

Sub. Code
23MMA1E5

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective – FUZZY SETS AND THEIR APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a membership function. Give an example.

2. Write down the algorithm for transitive closure $R_T(x, x)$.

3. Define a fuzzy measure with an example.

4. Write a short notes on the lattice of possibility distributions of length n.

5. What are the types of Hartley information?

6. Define a measure of confusion.

7. Draw a general scheme of a fuzzy controller.

8. What is meant by fuzzy dynamic system?

9. Define an individual decision making. Give an example.

10. Given an example for multicriteria decision problem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Compute the scalar cardinality and fuzzy cardinality for the following fuzzy sets:

(i) $C(x) = \frac{x}{x+1}$ for $x \in \{0,1,2,...,10\} = x$.

(ii) $D(x) = (1-x)/10$ for $x \in \{0,1,2,...,10\} = x$.

Or

(b) Explain the following types of fuzzy relations with an example for each:

(i) Reflexive; (ii) Symmetric;

(iii) Transitive; (iv) Asymmetric;

(v) Antireflexive.

12. (a) Write down the axioms of fuzzy measures. Also explain Dempster's rule of combination.

Or

(b) Let $x = \{a, b, c, d\}$. Given the basic assignment $m(\{a, b, c\}) = 0.5$, $m(\{a, b, d\}) = 0.2$ and $m(x) = 0.3$. Determine the corresponding belief and plausibility measures.

13. (a) Consider two fuzzy sets, A and B defined on the set of real number $X = [0, 4]$ by the membership grade functions $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$. Draw graphs for these functions and their standard classical complements.

Or

(b) With the usual notations, prove that $H(X, Y) \leq H(X) + H(Y)$.

14. (a) Explain the following defuzzification methods:

(i) Center of Area method;

(ii) Center of Maxima method;

(iii) Mean of Maxima method.

Or

(b) What is meant by fuzzy neural networks? Also write the basic features of the resulting networks.

15. (a) Explain multistage decision making with an illustration.

Or

(b) Let A be a symmetric trapezoidal type fuzzy number with ${}^{0+}A = [0, 4]$ and ${}^{1}A = [1, 3]$ and let B, C be symmetric triangular - type fuzzy numbers with centers $C_B = 4$, $C_C = 5$ and spreads $S_B = S_C = 2$. Rank these fuzzy numbers by each of the three ranking methods.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. What is meant by a compatibility relation? Explain with suitable illustration. Also determine the complete α-covers of the compatibility relation whose membership matrix is given below:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{pmatrix} 1 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0.8 & 1 & 0.7 & 0.5 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 & 1 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

17. Let $X = \{a, b, c, d, e\}$ and $Y = N_8$. Using a joint possibility distribution on $X \times Y$ given in terms of the matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0.3 & 0.5 & 0.2 & 0.4 & 0.1 \\ 0 & 0.7 & 0 & 0.6 & 1 & 0 & 0.4 & 0.3 \\ 0 & 0.5 & 0 & 0 & 1 & 0 & 1 & 0.5 \\ 1 & 1 & 1 & 0.5 & 0 & 0 & 1 & 0.4 \\ 0.8 & 0 & 0.9 & 0 & 1 & 0.7 & 1 & 0.2 \end{pmatrix} \end{array}$$

Determine the following:

(a) Marginal possibilities.

(b) Joint and marginal basic assignments.

18. State and prove the Gibbs' theorem.

19. Find a fuzzy automation with $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$, $Z = \{z_1, z_2, z_3, z_4\}$ whose output relations R and state transition relation S are defined, respectively, by the matrix.

$$R = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 0.3 \end{pmatrix} \end{array}$$

$$S = \left[\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} x_1 \\ \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{pmatrix} 0 & 0.4 & 0.2 & 1 \\ 0.3 & 1 & 0 & 0.2 \\ 0.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} x_2 \\ \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{pmatrix} \end{array} \right.$$

20. Let us assume that each individual of a group of eight decision makers has a total preference ordering $P_i (i \in \mathbb{N}_8)$ on a set of alternatives $X = \{w, x, y, z\}$ as follows :

$P_1 = \{w, x, y, z\}$, $P_2 = P_5 = \{z, y, x, w\}$,

$P_3 = P_7 = \{x, w, y, z\}$, $P_4 = P_8 = \{w, z, x, y\}$,

$P_6 = \{z, w, x, y\}$. Using the membership function $S(x_i, x_j) = \frac{N(x_i, x_j)}{n}$ for the fuzzy group performance, find the fuzzy preference relation. Also find α-cuts of this fuzzy relations S, and group level of agreement concerning the social choice denoted by the total ordering (w, z, x, y).

———————

S–3007

Sub. Code
23MMA1E6

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms:

   (a) Disjunctive normal form;

   (b) Conjunctive normal form.

2. What is meant by quantifiers? Give an example.

3. Write $p(x) = x^3 - 6x^2 + 11x - 6$ in telescopic form.

4. When will you say that a function is said to be recursive and partial recursive?

5. Let $X = \{1,2,3,4,6,12\}$ and $\leq$ be the usual less than or equal to relation. Draw the Hasse diagram of $(X, \leq)$.

6. Define the Boolean algebra with an example.

7. Define the Hamming distance. Give an illustration.

8. What do you mean by error correction for decoding function?

9. How many different ways are there to select 4 different players from 10 players on a team to play four tennis matches. Where the matches are ordered?

10. How many strings to length n can be formed from the English alphabet?

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) By not using the truth table directly, find PDNF for (i) $P \leftrightarrow Q$; (ii) $\neg P \vee Q$.

Or

(b) Verify the validity of the following inference :

"If one person is more successful than another, the he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is not more successful than Peter".

12. (a) Write the recurrence relation for Fibonacci numbers and solve it.

Or

(b) (i) Let $[\sqrt{x}]$ be the integral part of $\sqrt{x}$. Show that $[\sqrt{x}]$ is recursive.

(ii) If A denotes Ackermann's function, evaluate $A(1,2)$ and $A(3,1)$.

13. (a) If G is a group, then prove that the set of all normal subgroups of G forms a modular lattice.

Or

(b) Simplify the following using Karnaugh diagrams:

$f(x_1, x_2, x_3, x_4) = x_1x_3 + x_1'x_3x_4 + x_2x_3'x_4 + x_2'x_3x_4$.

14. (a) What is meant by coding theory? Draw block diagram for coding. Also list the uses of coding theory.

Or

(b) Define a group code with an example. Let $e : B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a non-zero code word.

15. (a) State and prove the pigeonhole principle. Also prove that among any $(n+1)$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.

Or

(b) What is the next largest permutation in lexicographic order after 362541?

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Using indirect method of proof, derive $P \to \neg S$ from $P \to Q \vee R$, $Q \to \neg P$, $S \to \neg R, P$.

(b) Write each of the following in symbolic form. (Assume that the universe consists of literally every thing)

(i) All men are giants

(ii) No men are giants

(iii) Some men are giants

(iv) Some men are not giants.

17. Solve the recurrence relation :

$S(k)-45S(k-1)-11S(k-2)+30S(k-3)=0$, $S(0)=0$, $S(1)=-35$, $S(2)=-85$.

18. (a) Show that every chain is a distribution lattice.

(b) Let L be a distribution lattice and $a,b,c \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then prove that $b = c$.

19. Suppose e is an (m,n) encoding function and d is a maximum likelihood decoding function associated with e. Prove that (e,d) can correct k or fewer errors if and only if the minimum distance of e is at least $(2k+1)$.

20. (a) Prove the following

(i) Pascal's identify;

(ii) Vandermonde's identity.

(b) Let n be a positive integer. Prove that $\sum_{k=0}^{n} C(n,k) = 2^n$

S–3008

Sub. Code
23MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

ADVANCED ALGEBRA

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define algebraic over a field F.

2. What do you mean by degree of finite extension and given an example.

3. State the Remainder theorem.

4. For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove that $[\alpha \cdot f(x)]' = \alpha \cdot f'(x)$.

5. Prove that $G(K,F)$ is a subgroup of the group of all automorphisms of K.

6. Define a normal extension and give an example.

7. State the Jacobson theorem.

8. Prove that the multiplicative group of non – zero elements of the finite filed is cyclic.

9. Define a solvable group.

10. Write the statement of Left–Division Algorithm.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the transitivity property of finite extensions.

Or

(b) Prove that if $a \in k$ is algebraic of degree n over F, then $[F(a) : F] = n$.

12. (a) Let $f(x) \in F[x]$ be of degree $n \geq 1$, then prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

13. (a) Let K be the field of complex numbers and F be the field of real numbers. Find $G(K,F)$ and $Fix[G(K,F)]$.

Or

(b) Write the statement for fundamental theorem of Galois theory.

14. (a) Prove that if F is a finite filed and $\alpha \neq 0, \beta \neq 0$ are two elements of F, then we can find elements 'a' and 'b' in F such that $1+\alpha \cdot a^2 + \beta \cdot b^2 = 0$.

Or

(b) Prove that any two finite fields having the same number of elements one isomorphic.

15. (a) Prove that G is solvable $\Leftrightarrow G^{(k)} = \{e\}$ for some integer K.

Or

(b) Prove that if $x \in H$, then $a^{-1} \in H$ if and only if $N(a) = 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the number e is transcendental.

17. Show that a polynomial of degree 'n' over a field F can have at most 'n' roots in any extension field.

18. Let K be the fixed field of $f(x)$ in $F[x]$, and let $p(x)$ be an irreducible factors of $f(x)$ in $F[x]$. If the roots of $p(x)$ are $\alpha_1, \alpha_2, ...\alpha_r$ then prove that for each i there exists an automorphism σ_i in $G(K,F)$ such that $\sigma_i\ (\alpha_i) = \alpha_i$.

19. Prove that a finite division ring is necessarily a commutative field.

20. Show that every positive integer can be expressed as the sum of squares of four integers.

———————

S–3009

Sub. Code
23MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Lebesque outer measure.

2. Show that if $F \in r$ and $m^*(F \Delta G) = 0$ then G is measurable.

3. Show that if f s integrable then f is finite valued a.e.

4. Define Riemann integrable over $[a, b]$.

5. State the Jordon's test.

6. Define Dirichlet's kernel.

7. Write first order Taylor formula.

8. State the mean value theorem.

9. Define saddle point of f.

10. Define Jacobian determined $J_f(x)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that for any sequence of sets $\{E_i\}$.

$$M^*\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} M^*(E_i).$$

Or

(b) Prove that every measurable set is not a boral set.

12. (a) State and prove Lebesgue's Dominated Convergence Theorem.

Or

(b) If f is a continuous on the finite interval $[a, b]$ then prove that f is integrable and $F(x) = \int_a^x f(t)dt \ (a < x < b)$ is a differentiable function such that $F'(x) = f(x)$.

13. (a) If $f \in L(I)$, then prove that, for each real β.

$$\lim_{\alpha\to\infty} \int_I f(t)\sin(\alpha t + \beta)dt = 0.$$

Or

(b) If $f \in L(-\infty + \infty)$, then prove that

$$\lim_{\alpha\to\infty} \int_{-\infty}^{\infty} f(t)\frac{1-\cos\alpha t}{t}dt = \int_0^{\infty} \frac{f(t)-f(-t)}{t}dt.$$

14. (a) If f is differentiable at c, then prove that f is continuous at c.

Or

(b) Let f and $D_2 f$ be continuous on a rectangle $[a, b]\times[c, d]$ let p, q be differentiable on $[c, d]$ where $p(y)\in[a, b]$ and $q(y)\in[a, b]$ for each y in $[c, d]$. Define F by the equation $F(y)=\int_{p(y)}^{q(y)} f(x, y)dx$ if $y\in[c, d]$, then prove that $F'(y)$ exists for each y in $[c, d]$ and is given by $F'(y)=\int_{p(y)}^{q(y)} D_2 f(x, y)dx +$ $f(q(y), y)\, q'(y) - f(p(y), y)\, p'(y)$.

15. (a) Let A be an open subset of R^n and $f : A \to \mathbb{R}^n$ has continuous partial derivatives $D_j F_i$ on A. If $J_f(x)\neq 0$ for all x in A, then prove that f is an open mapping.

Or

(b) A quadric surface with center at the origin has the equation

$$AX^2 + BY^2 + CZ^2 + 2DYZ + 2EZX + 2FXY = 1$$

Find the length of its semi axis.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that if $M^*(E) < \infty$ then E is measurable if and only if for all $E > 0$, there exists disjoint finite intervals $I_1, I_2, \ldots I_n$ such that $M^*\left(E \Delta \bigcup_{i=1}^{n} I_i\right) < \varepsilon$.

17. Prove that if f is Riemann integrable and bounded over the finite interval $[a, b]$, then f is integrable and $R\int_a^b f\,dx = \int_a^b f\,dx$.

18. Lef f be real valued and continuous on a compact interval $[a, b]$, then prove that for every $\in > 0$, there is a polynomial p such that $|f(x) - p(x)| < E$ for every x in $[a, b]$.

19. Assume that one of the partial derivatives $D, f, \ldots Dnf$ exists at C and that the remaining $n-1$ partial derivatives exists in some n-ball $B(C)$ and are continuous at C. Then prove that f is differentiable at C.

20. State and prove implicit function theorem.

S–3010

Sub. Code
23MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Write down the Telegraph equation.

2. Classify the partial Differential Equation $U_{xx} - 4U_{xy} + 4U_{yy} = e^y$.

3. State the Cauchy problem.

4. Write down the wave equation in spherical polar co-ordinates.

5. State the uniqueness theorem for heat conduction problem.

6. In the context of separability of partial Differential equations, mention the mixed boundary condition.

7. Differentiate between interior and exterior boundary value problems.

8. State the minimum principle for Harmonic functions.

9. Identify the Dirichlet problem involving the Laplace operators.

10. Define Dirac delta function.

Part B $(5 \times 5 = 25)$

Answer **all** the questions choosing either (a) or (b).

11. (a) Derive the one-dimensional wave equation for vibrating string.

Or

(b) Classify and transform the partial Differential equation $u_{xx} + u_{yy} + u_{xy} + u_x = 0$ into canonical form.

12. (a) Find the solution of the semi-infinite string with a free end.

$$u_{tt} = c^2 u_{xx}, 0 < x < \infty, t > 0$$
$$u(x,0) = f(x), u_t(x,0) = g(x), 0 \le x < \infty .$$
$$u_x(0,t) = 0, 0 \le t < \infty$$

Or

(b) Obtain the solution of the initial value problem

$$u_{tt} = c^2 u_{xx} \text{ with } u(x,0) = \sin x, u_t(x,0) = \cos x$$

13. (a) State and prove the Existence theorem for one-Dimensional Heat equation.

Or

(b) Determine the solution of the initial-boundary value problem: $u_{tt} - u_{xx} = h$, $0 < x < 1$, $t > 0$, h-constant $u(x,0) = x(1-x)$, $0 \le x \le 1$, $u_t(x,0) = 0$, $0 \le x \le 1$ $u(0,t) = u(1,t) = 0$, $t \ge 0$.

14. (a) Show that the solution of a Dirichlet problem if exists is unique and depends continuously on the boundary data.

Or

(b) Solve the Torsion problem given below.

$$\nabla^2 u = -2, 0 < x < a; 0 < y < b$$
$$u(0, y) = u(a, y) = 0, 0 \le y \le b$$
$$u(x,0) = u(x,b) = 0, 0 \le x \le a$$

15. (a) Determine the solution of the Dirichlet problem $\nabla^2 u = h$ in D and $u = f$ on B by the method of Green's function.

Or

(b) Define Green's function and show that it is symmetric.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the general solution of the partial Differential Equation $y^2 u_{xx} - x^2 u_{yy} = 0$, by its canonical form.

17. Derive the D' Alembert solution of the Cauchy problem for the one dimension wave equation.

18. Using the separation of variables technique, solve the Heat conduction problem

$$u_t = k u_{xx}, 0 < x < l, t > 0$$
$$u(0,t) = u(l,t) = 0, t \geq 0$$
$$u(x,0) = x(l-x), 0 \leq x \leq l$$

19. Obtain the solution of the Dirichlet problem for a rectangle.

20. Determine the solution of the Dirichlet problem for unit circle by the method of Green's function.

———————

S–3011

Sub. Code
23MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective — ALGEBRAIC GEOMETRY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define radical of an ideal.

2. Define Blow-up algebra.

3. Define Jacobson ring.

4. Define Normal domains.

5. Define affine variety.

6. State Krull dimension theorem.

7. State universal property of products.

8. Define Dominant morphisms.

9. State segre embedding.

10. Define projective change of co-ordinates.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let f be a non-constant polynomial map. Prove that f is of degree $r\ (\geq 1)$ if and only if $\Delta(f)$ is of degree $r-1$.

Or

(b) State and prove Krull's principle ideal theorem.

12. (a) Prove that a noetherian domain R is normal if and only if

(i) R_p is a DVR for every prime ideal p of height 1 and

(ii) $R = \bigcap_{hk(p)=1} R_p$.

Or

(b) Prove that a field k which is finitely generated over a perfect field k is separable generated.

13. (a) Prove that an algebraic subset V of $\subseteq \mathbb{A}^n$ is a hyper surface $\Leftrightarrow$ V is pure codimension 1 in $\mathbb{A}^n$.

Or

(b) Prove that an algebraic subset $V \subseteq \mathbb{A}^n$ is irreducible if and only if its ideal I (V) is a prime ideal of R (n). In particular $\mathbb{A}^n$ is irreducible.

14. (a) Let $\phi : V \to W$ be a morphism and ϕ^* be the induced homomorphism of k-algebras. Prove that the following :

(i) ϕ is an isomorphism onto a closed subset of $W \Leftrightarrow \phi^*$ is surjective.

(ii) ϕ is dominant $\Leftrightarrow \phi^*$ is injective.

Or

(b) Prove that a variety V of dimension d is birational to some irreducible hyper surface in $\mathbb{A}^{d+1}$. In particular a curve is birational to a plane curve.

15. (a) State and prove Dimension of a cone.

Or

(b) State and prove Veronese Embedding theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove Nakayama Lemma.

17. Let B be an integral domain which is also an affine algebra over a field k. Let C be an affine subalgebra of B such that dim (C) = dim (B) and C is integrally closed in B. Let $m \in$ (Max B) be such that it is minimal among the prime ideals of B lying over $p = cnm$. Prove that $C_p = B_m$.

18. Let $V \subseteq \mathbb{A}^n$ and $k[V]$ be the coordinate ring of V. Prove the following :

(a) The set V is bijective with $Max\ (k[V])$.

(b) The set of closed subsets $z(a)$ of V are bijective with the set of radical ideals a of $k[V]$.

(c) The principal affine open subsets $D(f)$, $f \in k[V]$, form a base of open sets for the topology of V.

(d) V is connected $\Leftrightarrow k[V]$ has no idempotents other than 0 and 1.

19. State and prove Hausdroff Axiom.

20. State and prove Dimension of Inter sections theorem.

S–3012

Sub. Code
23MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

***Elective* — MATHEMATICAL STATISTICS**

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that the probability of the nulls set is zero.

2. Define a random variable. Give an example.

3. Define moment generating function.

4. Let the joint pdf of X_1 and X_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \textit{elsewhere.} \end{cases}$$

Show that X_1 and X_2 are dependent.

5. Define a negative binomial distribution.

6. Find the moment generating function of a Poisson distribution.

7. Define the *F*-distribution.

8. When will you say that X_n is said to be converges in probability to X?

9. Let $X_1,...,X_n$ be a random sample from the $\Gamma(2, \theta)$ distribution, where θ is unknown. Let $Y = \sum_{i=1}^{n} X_i$. If $n = 5$, show that $P\left(9.59 < \frac{2Y}{\theta} < 34.2\right) = 0.95$.

10. Give an example for estimation of π by Monte carlo integration.

Section B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Bonferroni's inequality.

Or

(b) Let us select five cards at random and without replacement from an ordinary deck of playing cards.

(i) Find the pmf of X, the number of hearts in the five cards

(ii) Determine $P(X \le 1)$.

12. (a) Let X have the pdf $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \textit{elsewhere.} \end{cases}$ Find $E(X), E(X)$ and $E(6X + 3X^2)$.

Or

(b) Let X_1 and X_2 have the joint pmf $p(x_1, x_2) = x_1 x_2 / 36$. $x_1 = 1, 2, 3$ and $x_2 = 1, 2, 3$, zero elsewhere. Find first the joint pmf of $Y_1 = X_1 X_2$ and $Y_2 = X_2$, and then find the marginal pmf of Y_1.

13. (a) Let X have a poisson distribution with $\mu = 100$. Use Chebyshev's inequality to determine a lower bound for $p(75 < X < 125)$.

Or

(b) Define Gamma p.d.f. Obtain its moment generating function. Deduce its mean and variance.

14. (a) State and prove Weak law of large numbers theorem.

Or

(b) State and prove Central limit theorem.

15. (a) Let $X_1, X_2, ..., X_n$ be a random sample from the Bernoulli distribution, $b(1, p)$, where p is unknown.

Let $Y = \sum_{i=1}^{n} X_i$.

(i) Find the distribution of Y

(ii) Show that $\frac{Y}{n}$ is an unbiased estimator of p

(iii) What is the variance of $\frac{Y}{n}$?

Or

(b) Let the observed value of the mean $\overline{X}$ of a random sample of size 20 from a distribution that is $N(\mu, 80)$ be 81.2. Find a 95 percent confidence interval of μ

Section C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) For any random variable, prove that $P[X = x] = F_X(x) - F_X(x-)$, for all $x \in R$, where .

(b) Given $\int_C \left[\frac{1}{\pi(1+x^2)}\right] dx$ where $C \subset \mathcal{C} = \{x : -\infty < x < \infty\}$. Show that the integral could serve as a probability set function of a random variable X whose space is $\mathcal{C}$.

17. (a) Derive the Markov's inequality.

(b) State and prove the Chebyshev's inequality.

18. (a) If the random variable X is $N(\mu, \sigma^2), \sigma^2 > 0,$ then prove that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

(b) If X is $N(\mu, \sigma^2),$ show that $E(|X-\mu|) = \sigma\sqrt{\frac{2}{\pi}}$.

19. Establish Student's t-distribution.

20. Fit a Poisson distribution to the following data :

x	0	1	2	3	$3 < x$
Frequency	20	40	16	18	6

(a) Compute the corresponding chi-square goodness-of-fit statistic

(b) How many degrees of freedom are associated with this chi-square?

(c) Do these data result in the rejection of the Poisson model at the $\alpha = 0.05$ significance level.

S–3013

Sub. Code
23MMA2E3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective — TENSOR ANALYSIS AND RELATIVITY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** the questions.

1. What is meant by Tensor? Give an example.

2. Define the mixed tensor with an example.

3. Define the metric tensor.

4. State the Riemann – Christoffel tensor.

5. Write down the Bianchi identify.

6. Show that $\delta_{ijk}^{ijk} = 3!$ if $i, j, k = 1,2,3$.

7. What is meant by Galilean transformation?

8. Define the proper time and proper distance of the particle.

9. Define the following terms :

(a) Relativistic momentum;

(b) Proper mass.

10. What is meant by longitudinal mass and transverse mass?

Section B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Discuss the transformations in which the coordinates y^i are rectangular cartesian :

$$y^1 = x^1 \cos x^2,\ y^2 = x^1 \sin x^2, y^3 = x^3$$

Or

(b) If a_{ij} is a tensor, show that A^{ij}, the cofactor of a_{ij} in $|a_{ij}|$ divided by $|a_{ij}|$ is a tensor.

12. (a) Let E_3 be covered by orthogonal cartesian coordinates x^i, and consider a transformation $x^1 = y^1 \sin y^2 \cos y^3$, $x^2 = y^1 \sin y^2 \sin y^3$, $x^3 = y^1 \cos y^2$, where the y^i are spherical polar coordinates $(y^1 = r,\ y^2 = \theta, y^3 = \varphi)$ the metric coefficients $g_{ij}(y)$.

Or

(b) Derive the Christoffel symbols of the second kind.

13. (a) State and prove the Einstein tensor.

Or

(b) Prove that the covariant derivatives of generalized Kronecker deltas vanish identically.

14. (a) Narrate the Ether theory.

Or

(b) Explain the following terms :

(i) The world time;

(ii) The twin paradox.

15. (a) With the usual notations, prove that the Hamiltonian function $H(x,p,t) = c\sqrt{m_0^2c^2 + p_1^2 + p_2^2 + p_3^2} + V(x,t)$.

Or

(b) Suppose a round trip is to be made by rocket from the earth to a near by star, Alpha centauri, which is about 4 light-years distant. The rocket is capable of a constant acceleration $g = 9.50\ \text{m/sec}^2$ $(1\,lt - yr/yr^2)$ relative to its momentary rest frame. What is the time required for the trip?

Section C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If $f(x^1, x^2, \ldots x^n)$ is a homogeneous function of degree m, then prove that $\frac{\partial f}{\partial x^i} x^i = mf$.

(b) Prove that the sum (or difference) of two tensors which have the same number of covariant and the same number of contravariant indices is again a tensor of the same type and rank as the given tensors.

17. (a) State and prove the Ricci's theorem.

(b) Show that $\frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i} = [jk, i] - [ij, k]$.

18. Show that a necessary and sufficient condition that the metric coefficients $g_{ij}(x)$ reduce to constants h_{ij} in some reference frame y is that the christoffel symbols $_y\left\{ \begin{matrix} k \\ ij \end{matrix} \right\}$ vanish identically.

19. (a) A particle moves relative to the frame I' with a velocity v' in a direction given by the angle φ' measured from the positive x' axis. Find the amplitude and direction of the velocity of this particle relative to the I frame.

(b) Enumerate the relativistic Doppler Effect.

20. Discuss in all details of Rocket with constant thrust.

———————

S–3014

Sub. Code
23MMA2E4

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective – CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define variation of a functional.

2. State the fundamental lemma of the calculus of variation.

3. What is the transversality condition?

4. Define external field.

5. What is an isoperimetric problem?

6. What is the Ritz method in calculus of variations?

7. Define singular integral equation.

8. What do you mean by homogeneous integral equations?

9. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 e^{s-t}.g(t)\, dt$.

10. State Fredholm's second theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Solve the Brachistochrone problem.

Or

(b) Find the extermals of the functional

$$v[y(x),z(x)] = \int_0^{\frac{\pi}{2}} [y'^2 + z'^2 + 2yz]\, dx, y(0) = 0;\ y\left(\frac{\pi}{2}\right) = 1;$$

$$z(0) = 0,\ z\left(\frac{\pi}{2}\right) = -1.$$

12. (a) Find a solution with one corner point is the minimum problem of the functional

$$v[y(x)] = \int_0^4 (y'-1)^2 (y'+1)^2\, dx;\ y(0) = 0;\ y(4) = 2.$$

Or

(b) Is the Jacobi condition fulfilled for the extermal of the function $v = \int_0^a (y'^2 - y^2)\, dx$ that passes through the points $A(0,0)$ and $B(a,0)$?

13. (a) Find the extermals of the isoperimetric problem $v[y(x)] = \int_0^1 (y'^2 + x^2)\, dx$ given that $\int_0^1 y^2\, dx = 2;$

$y(0) = 0;\ y(1) = 0.$

Or

(b) Test for an extremum the functional

$$v = \int_0^1 (ax^3 y''^2 - bxy^2)\, dx;\ y(1) = y'(1) = 0$$ where a and b are positive constants.

14. (a) Explain briefly the types of kernels through an example.

Or

(b) Solve the fredholm integral equation of the second kind $g(s) = s + \lambda \int_0^1 (st^2 + s^2 t)\, g(t)\, dt$.

15. (a) Find the Neumann series for the solution of the integral equation $g(s) = (1+s) + \lambda \int_0^s (s-t)\, g(t)\, dt$.

Or

(b) Evaluate the resolvent for the integral equation

$$g(s) = f(s) + \lambda \int_0^1 (s+t)\, g(t)\, dt .$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Derive Euler - poisson equation.

17. Test for an extremum the functional $\int_0^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$ given that $y(0) = 0, y_1 = x_1 - 5$.

18. Find the form of an absolutely flexible, non extensible homogeneous rope of length and suspended at the points A and B.

19. Solve the integral equation

$$g(s) = f(s + \lambda) \int_0^1 (1 - 3st)\ g(t)\ dt\ .$$

20. Describe the solution of volterra integral equation by the method of successive approximations.

S–3015

Sub. Code
23MMA2E5

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective – WAVELETS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Discrete Fourier transform.
2. Define Discrete delta function.
3. Define Decimation operator.
4. What is biorthogonal wavelets?
5. Prove that the trigonometric systems in complete in $L^2([-\pi,\pi])$.
6. What is square integrable?
7. Define the inverse Fourier transform on $L^2([-\pi,\pi])$.
8. Define convolution of z and w.
9. What is an Approximate identity?
10. State the Lebesgue dominated convergence theorem.

Section B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that if $z, w \in l^2(\mathbb{Z}_N)$ then for each M, $(z * w)^\wedge(M) = \hat{z}(M)\hat{w}(M)$.

Or

(b) Prove that if $N = 2^n$ for some $n \in N$ then $\#_N \leq \frac{1}{2} N \log_2{}^N$.

12. (a) Suppose N is even, say $N = 2M, Z \in l^2(\mathbb{Z}_N)$ and $x, y, w \in l^2(\mathbb{Z}_{N/2})$ then prove that $D(z) * w = D(z * U(w))$,

$U(x) * U(y) = U(x * y)$.

Or

(b) State and prove the folding lemma.

13. (a) Suppose H is a Hilbert space, $\{aj\}_{j \in z}$ is an orthonormal set in H, and $z = (z(j))_{j \in z} \in l^2(\mathbb{Z})$ then prove that the series $\sum_{j \in z} (z(j)) a_j$ converges in H, and $\left\| \sum_{j \in z} z(j) a_j \right\|^2 = \sum_{j \in z} |z(j)|^2$.

Or

(b) Suppose H is a Hilbert space and $\{aj\}_{j \in z}$ is an orthonormal set in H. Then prove that $\{aj\}_{j \in z}$ in a complete orthonormal set If and only if $f = \sum_{j \in z} < f, a_j > aj$ for all $f \in H$.

14. (a) Suppose $z \in l^2(\mathbb{Z})$ and $w \in l^1(\mathbb{Z})$. Then prove that $z * w \in l^2(\mathbb{Z})$ and $\|z * w\| \le \|w\|, \|z\|$.

Or

(b) Suppose $T : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ is a bounded translation-invariant linear transformation. Define $b \in l^2(\mathbb{Z})$ $b = T(\delta)$ then prove that, for all $z \in l^2(\mathbb{Z})$, $T(z) = b * Z$.

15. (a) State and prove the Parseval's relation and Plancherel's formula.

Or

(b) Suppose $\varphi \in L^2(\mathbb{R})$, and for each $j \in (\mathbb{Z})$, $\{\varphi_j, k\}_{k \in \mathbb{Z}}$ is an orthonormal set. Define $\{v_j\}_{i \in z}$ by equation $v_j = \left\{\sum_{k \in z} z(k)\varphi_{j,k} : z = (z(k))_{k \in z} \in l^2(\mathbb{Z})\right\}$ then prove that $\bigcap_{j \in z} v_j = \{0\}$

Section C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If $T : l^2(\mathbb{Z}_N)$ is a translation-invariant linear transformation. Then prove that each element of the Fourier basis F is an eigen vector of T in particular T is diagonalizable.

17. State and prove a uniqueness result of Fourier series.

18. Suppose $M \in \mathbb{N}$, $N = 2M$, and $w \in l^2(\mathbb{Z}_N)$ then prove that $\{R_{2k}w\}_{k=0}^{M-1}$ is an orthonormal set with M elements if and only if $|\hat{w}(n)|^2 + |\hat{w}(n+M)|^2 = 2$ for $n = 0, 1, \ldots M-1$.

19. If $u, v \in l'(\mathbb{Z})$. Then prove that $B = \{R_{2k}v\}_{k\in\mathbb{Z}} \cup \{R_{2k}u\}_{k\in\mathbb{Z}}$ is a complete orthonormal set in $l^2(\mathbb{Z})$ if and only if the system matrix $\Lambda(\theta)$ is unitary for all $\theta \in [0, \pi)$.

20. Suppose $f \in L^1(\mathbb{R})$ and $\{g_t\}_{t>0}$ is an approximate identity. Then prove that, for every Lebesque point *x* of *f*, $\lim_{t\to 0^+} g_t * f(x) = f(x)$.

S–3016

Sub. Code
23MMA2E6

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective — MACHINE LEARNING AND ARTIFICIAL INTELLIGENCE

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is Machine Learning?

2. Define Version Spaces.

3. What is Back Propagation?

4. Define Gradient Decent.

5. What is Maximum likelihood hypothesis?

6. What is meant by belief network?

7. Define an Agent.

8. Define Maximum expected utility.

9. What is Inference?

10. What are the components of a planning system?

Part B (5 × 5 = 25)

Answer **all** questions by choosing either (a) or (b).

11. (a) What are the issues in machine learning?

Or

(b) Explain the concept of Inductive Bias.

12. (a) Derive the Gradient Descent rule.

Or

(b) Explain the concept of Genetic programming.

13. (a) Discuss about expectation maximization (EM) algorithm.

Or

(b) Explain Naive Bayes classifier with an example.

14. (a) Explain a simple reflex agent with a diagram.

Or

(b) Specify the syntax of First-order logic in BNF form.

15. (a) Define the following terms :

(i) Prior probabilities

(ii) Conditional probabilities

(iii) Full joint probability distribution

Or

(b) Write backward space search algorithm and lifted backward state space search algorithm.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain Find S algorithm with given example. Give its application.

Example	Sky	Air Temp	Humidity	Wind	Water	Forecast	Enjoy sport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

17. Derive the Back propagation rule considering the training rule for output unit weights and training rule for hidden unit weights.

18. Describe maximum likelihood hypothesis for predicting probabilities.

19. Explain in detail the connectives used in propositional logic.

20. Explain Heuristics for planning.

———————